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Professor Laurence Goldstein

Department of Philosophy

"Paradoxes shallow and deep"

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Paradoxes Shallow and Deep

Laurence Goldstein

Vice-chancellor, ladies and gentlemen

Acknowledgments

To assume the Chair of Philosophy at Swansea is not to assume the Chair of *any* department. The Department has an extremely distinguished history; indeed it used to be the flagship bearer of the College. The distinctive style of philosophizing prosecuted here became widely known as `the Swansea tradition'. More recently – some eight years ago -- the Department again received widespread attention when it was the centre of a furious and acrimonious dispute which ramified into controversy over intellectual standards, academic freedom and the relation of the teacher to the College. Many interesting questions worthy of much serious debate came to the fore, though discussion of them at the time may have sometimes been clouded by rage and tears. They have exercised me considerably since coming here and might have been the focus of my talk tonight but for the fact that the issues are so complex that I have hardly begun to think them through, and would certainly not be able to do justice to them in the fifty minutes allotted to me.

At the centre both of the Swansea tradition and of the aforementioned controversy was my predecessor in this Chair, Professor D.Z. Phillips. At his own inaugural in 1971, D.Z. had the sombre duty of paying tribute to his predecessor, Professor J.R. Jones, who had died one year earlier. It is my good fortune and honour to be succeeding an eminent philosopher who is very much alive and (some say) kicking. Talking, certainly. D.Z has a rare passion both for talking philosophy and for teaching the subject, an irrepressible enthusiasm that is almost unnerving. He continues to lecture here even though he took an early retirement to make time for editing the posthumous works of a hero of the Swansea tradition in philosophy, the Wittgenstein scholar Rush Rhees.

Reverting, for a minute, to the sombre, I should like to say a few words about Richard Beardsmore who had been Head of the Department of Philosophy and who died at a tragically early age. I never had the pleasure of meeting Dick, but people both within and outside the College have been eager to tell me about him. I have several times heard him referred to as a `star', this term interpreted both in the vernacular sense of `a great guy', but also in the sense of his being a luminous intellect. Next year the Department has its seventy fifth birthday, and part of the celebration of this anniversary will be to publish a book that Dick had all but finished at the time of his death.

Introduction

In his own inaugural address `Some Limits to Moral Endeavours', D.Z. Phillips discusses moral dilemmas for which, he alleges, there are no solutions. In his view, the failure to recognize that there are sometimes no recipes for successful resolution leads to a diminution of our sensitivity, of our sense of tragedy, and, correlatively to an erosion of care and compassion. Ilham Dilman, in the inaugural he gave to mark the award of his personal chair, also pursues the theme of impossibility – `the impossibility of *your* taking *my* decisions, facing *my* difficulties, feeling *my* distress, loving or dying in *my* place' (p.7) and of the recognition of this separation as making impossible, at least, according to Marcel Proust, the uniting of one's life with that of another. If these are indeed genuine examples of impossibilities, then they are *surprising* and that they are

indeed impossible needs to be demonstrated in order to be believed. Proofs of surprising impossibilities are generally of great interest.

My topic for this evening is also that of impossibility – the impossibility (or alleged impossibility) of solving certain problems. In mediaeval discussions, the name for one class of such problems was *insolubilia* (unsolvables). What I hope to show this evening is that certain of the problems labelled *insolubilia* are, while difficult, not *impossible* to solve. A better label for them would be *intractabilia*. These are problems that, while they may have been thought to be impossible to solve are demonstrably not so. The modern name for an *insolubile* is a *paradox*.

There are paradoxes in many areas of enquiry, including logic, semantics, physics, metaphysics.and behavioural studies. We may characterize a paradox as an argument which begins with premises that seem entirely acceptable and which proceeds via steps of inference that seem perfectly valid, to a conclusion which is contradictory or apparently absurd. That we reach such a conclusion is *para doxa* – literally, beyond belief – or is at least highly surprising. The importance of paradoxes is, paradoxically, that they are generally so simple. If simple steps of reasoning lead from plausible assumptions to absurdity, that indicates that there is something wrong either with some of our basic assumptions or with some of our fundamental principles of reasoning, so the investigation of paradoxes goes right down to the foundations of our thinking.

Shallow and Deep

My strategy will be to show that there are connections between some of the traditional deep paradoxes and shallower, more recreational, less intractable ones. The key to solving the deep paradoxes is to see that nothing essentially problematic is introduced in the transition from the shallow to the deep. In what follows, I shall make several transitions from the shallow to the deep, and shall employ a number of simple techniques for doing so.

To give you the flavour of what I mean by a shallow paradox, consider a man who goes into a bookstore and asks the saleswoman 'Where is the self-help section?' The saleswoman replies that it would defeat his purpose if she revealed the answer. This is funny rather than deep because the man was clearly interested in books about do-it-yourself or about medical self-diagnosis, and not about books on finding one's way around a new bookstore. It would hardly be fair to call so silly a story a paradox. However, with a little tweaking of the parameters, we can get a problem that begins to look genuinely paradoxical. Consider, for example, a woman going into a psychiatrist and, to the psychiatrist's traditional enquiry 'How can I help?', she replies 'The only help you can give me is not to give me any help'. Here, if the psychiatrist complies with her wishes and gives her no help then, in so doing, he is giving her help of just the sort she mentions. On the other hand if he gives her help then, if she is right, that is no help at all. It seems that the psychiatrist can take one of two courses of action – either giving or not giving help – but either way, he seems to end up, impossibly, both helping and not helping the woman. We can extricate ourselves from this difficulty, however, by seeing the argument as a *reductio ad absurdum*, a proof that the woman's reply to the psychiatrist was simply false. So, if the puzzle is called a paradox, it belongs at the shallow end.

What, then, is a deep paradox? It is a paradox that has proved recalcitrant – highly resistant to solution. Some paradoxes which have been tackled by some of the greatest minds of the past two millenia have remained unsolved to this day. Perhaps the best known of these is the Liar Paradox which features an individual who says 'This very statement that I am now making is not true'. The speaker is referring to the very statement he is in the process of making. Though

not usual, such self-reference is not generally impossible. I may say, for example: 'This statement is not very interesting, and may be the last one you hear before nodding off'. Unproblematically true. By contrast, there is a problem with saying that the Liar statement is true, for, if it is true, what it says is how things are, and what it says is that it is *not* true. On the other hand, if it is not true, then that's just what it says itself to be, so it is true. (There are, of course, more formal, rigorous demonstrations of this contradiction.)

Another deep, ancient paradox, is the Sorites – the Paradox of the Heap. This turns on the idea that, if you place one grain of sand on the ground, it does not constitute a heap. Add another – still no heap. Add another..... In general, if you have n grains of sand that do not constitute a heap then the addition of one more will not produce a heap. The paradoxical conclusion is that after adding a million grains, you still have no heap -- yet patently you do.

Yet another deep paradox is a variant of one of the paradoxes of motion due to Zeno of Elea. Since some people think that Zeno's paradoxes are solved by the mathematical theory of infinitesimals, I choose a variant due to José Bernadete[1] which is not susceptible to this treatment.

> A man decides to walk one mile from A to B. A god waits in readiness to throw up a wall blocking the man's further advance when the man has travelled ½ mile. A second god (unbeknown to the first) waits in readiness to throw up a wall of his own blocking the man's further advance when the man has travelled ¼ mile. A third god … &c. *ad infinitum*. It is clear that this infinite sequence of mere intentions (assuming the contrary-to-fact conditional that each god would succeed in executing his intentions if given the opportunity) logically entails the consequence that the man will be arrested

> at point A; he will not be able to pass beyond it, even though not a single wall will in fact be thrown down in his path. The ... [effect] will be described by the man as a strange field of force blocking his passage forward.

Of course, all this talk of gods throwing up walls is purely for dramatic effect – we could state the paradox in a more sober fashion. And it is a tough paradox – it has led at least one author (Priest, *op. cit.*) to conclude that motion may be contradictory.

One final example of a deep paradox – this one, though relatively youthful, is soon to celebrate its centenary. The paradox was discovered by Bertrand Russell. The assumptions on which it rests seem as plain as the nose on your face. Consider the set of days of the week. This contains just seven members, and does not contain itself; it contains the days of the week and does not contain any sets. So the set of days of the week does not contain itself as a member. But there are sets which do contain themselves as members – for example, the set of things that have never won an Olympic Gold medal. This set (unfortunately) contains me, and it also contains itself, since the set of things that have never won an Olympic Gold is itself one of the many individuals that have never won an Olympic Gold. We see, then, that there are some sets that are self-membered, others that are non-self membered. Now let us consider assembling a set – call it the Rusell set, or R for short, – which contains as its members all and only the non-self-membered sets. Here's the question: Is the Russell set a member of itself? We can easily state the condition for some set being a member of the Russell Set:

x is a member of R if and only if x is not a member of x

To see whether R is a member of R, plug R in for x in this condition. The surprising outcome is that R is a member of itself if and only if R is not a member of itself. So, assuming that it is either a member of itself or not, we conclude that it both is and is not a member of itself – an absurdity.

First Transition – from The Barber to Russell

A story is told about the village of Alcala and the barber in that village who shaves all and only those who do not shave themselves. Does this barber shave himself? My picture of Alcala is of a sleepy settlement in which at least twenty adult males dwell, most too listless to shave themselves. Now imagine that bubonic plague takes the lives of all the male inhabitants, sparing only the barber. Well, he shaves exactly the persons who don't shave themselves, and he himself is the only person surviving who needs a shave. So he shaves himself if and only if he doesn't shave himself ......clearly nobody could perform such a feat. This illustrates a manoeuvre which is a very useful one for dealing with paradoxes: reduce to the simplest case, ensuring that nothing vital is lost in the reduction. We cut out the 'noise' – in this case the other male villagers. We can imagine their number shrinking to two, one or zero and the shape of the problem remains the same. I'll call this technique *shrinking*. No barber could exist who both shaves and does not shave himself and, only slightly less obviously, no barber can exist who shaves all and only those villagers who do not shave themselves. For the bottom line will be that he shaves himself if and only if he does not. Thus, the answer here, as in the case of the psychiatrist's patient we considered earlier, is that the starting assumption is false. That assumption was that there *is* a barber conforming to the stated description. By *reductio*, there is no such barber. In other words, we specify no barber by the condition 'he who shaves all and only those who do not shave themselves'. This is generally accepted, hence it is now generally accepted that there is no deep paradox about the eponymous Barber.

Now, compare the Barber with the Russell Paradox. They have a common stucture – they are of the same form. This can be seen by comparing the specification for the Russell Set with that of the Barber:

x is shaved by B if and only if x is not shaved by x

So just as we are happy to say that there is no barber specified in this way, should we not be equally happy to say that there is no Russell Set? The answer is `no' because we can see absolutely no reason for denying that all the non-self-membered sets can be assembled into a set. Sets, unlike barbers, are not subject to the contingencies of physical existence.

There are two directions to take at this point. One is to abandon at least one of the axioms of our intuitive, naïve set theory and to provide instead axioms which, while plausibly capturing the notion of a set, are such that they are consistent – do not lead to any contradiction. Another direction is to take seriously the analogy with the Barber, accept the consequence that the Russell Set does not exist and try to free ourselves of the deep-rooted preconception that it must. Our intuition that the barber of Alcala must exist is relatively easy to shift, but, if the analogy with the Russell Paradox is a good one, we might be able to free ourselves of the belief in the Russell Set too. The *method of analogy* is another important technique for moving from the deep to the shallow, just so long as the analogy in question is compelling and not far-fetched. We may be inclined to suppose, for example, that there is some sort of analogy between the Liar and certain of the impossible objects of the graphic artist Maurits Escher, such that an understanding of the

etchings aids understanding of the Liar. While I think this is true, it is obvious that the analogy needs to be spelled out in great detail if it is to be persuasive.[2] Our belief that the Russell Set must exist rests, I belive, on a *poor* analogy between a set and a container. We imagine ourselves taking non-self-membered sets like the set of all horses, the set of prime numbers and shoving them into the container set R. It should be clear, however, that this analogy is a poor one if only because there are sets which include among their members those sets themselves, whereas no physical container can contain itself. Sets are mathematical entities defined by non-contradictory membership specifications. The specification for the Russell Set is a contradiction.[3]

Second Transition – From Munchausen to the Surprise Examination

It is time now to return to the psychiatrist's couch. Some of you may have heard of Munchausen's Syndrome; some of you may even suffer from it. If a person fakes an illness that he has no evidence to believe that he is suffering from, but does so merely to draw attention to himself and elicit sympathy, then that person is suffering from Munchausen's Syndrome. So if you secretly draw red blotches on your body with lipstick but tell people that you have measles then, if this is compulsive behaviour and not just a prank, more than likely you have Munchausen's – the measles strain of Munchausen's. Notice that it would be quite possible to have the cancer strain of Munchausen's and actually have cancer – if, for example, the cancer was at an early stage and you had no evidence that you had contracted it, but you faked having it. Now suppose that a woman went into a psychiatrist and, in full view of the psychiatrist, started painting red blotches on her body with lipstick. What should we say of this person? That she is pretending to have Munchausen's Syndrome; that she has the Munchausen strain of Munchausen's; that she has meta-Munchausen's? Can a person fake Munchausen's Syndrome?[4] If someone goes to a doctor and says 'I have measles', while believing that she doesn't, then that person has Munchausen's. But what if a person went to a doctor and said 'I have Munchausen's', while believing that she doesn't? We may be inclined to conclude that she has Munchausen's, for she is claiming to have a syndrome that she believes she does not have. On the other hand, the woman herself would reach that same conclusion, so she would believe that she *has* Munchausen's. Hence, in announcing that she has it, she is claiming to have a disease that she believes she has, so there is no faking, hence *no* Munchausen's. Thus, if someone has Munchausen's then we can say of her 'She has Munchausen's' but, as we have seen, there is some kind of absurdity involved in her making the first-person report '*I* have Munchausen's'.

There are simpler cases of this kind of disparity between third- and first-person reports. You can say of me 'He has forgotten that his name is 'Laurence'' but obviously I could not sincerely say 'I have forgotten that my name is 'Laurence''. Or take hypochondria. Clearly I can truly ascribe it to someone by saying 'He believes he's suffering a disease, but he isn't', yet it would be bizarre to report in the first person 'I believe that I am suffering a disease but I am not'. As Leslie Stevenson says, 'there is something strongly irrational, indeed unintelligible, about *representing oneself* either as believing something while simultaneously asserting it to be false, or as not believing something while asserting it as true'.[5] This cannot be the whole story, however, because surely one can accept the testimony of a physician who diagnoses oneself as having hypochondria, and accordingly ascribe it to oneself. This illustrates the point that the locution 'I believe that p' is used simply as the expression of a belief p, but is also sometimes used to report the state of mind of the believer, a state about which another person may sometimes have better knowledge than the subject himself.

Statements of the form 'I believe p, but p is not true' are termed Moore-paradoxical (or, more briefly Mooronic) after the Cambridge philosopher G.E. Moore. The paradox is that we, as commentators, can see that both halves of such a statement could be true (i.e., p could be true even though that proposition is not believed by the speaker) yet the first-person utterance by the speaker is absurd. Why should it be absurd to say what may be true? In the case where the utterance of 'I believe p' is an *expression* of the belief that p, the answer is relatively strightforward: the absurdity involved is just the absurdity of uttering a contradiction.

With these observations in place, let us look now at the Paradox of the Surprise Examination which, in various guises, has existed for over fifty years. Recall the situation: A teacher, on Friday evening, says to her class 'There will be a surprise examination one day next week'. The pupils figure out immediately that next Friday is not a possible day for a surprise examination since, if no exam has taken place by Thursday evening, Friday would be the *only* available day, and hence could not be a surprise. After a little further thought, the pupils figure out that Thursday is also not a possible day for a surprise exam since, with Friday now eliminated, if no exam has taken place by Wednesday evening, Thursday would be the only available day, hence an exam given on that day could be no surprise. But now, of course, a similar pattern of reasoning eliminates Wednesday, Tuesday and Monday and hence leads the pupils to conclude that no surprise examination can take place. They are therefore surprised when, one day the following week, the teacher comes in and starts handing out the examination paper.

Where has the pupils' reasoning gone wrong? Many solutions have ben proposed. A promising approach, I believe, is first to shrink the problem, eliminating the noise of the five-day week, and have the teacher announce to her class: There will be a surprise examination *tomorrow*. What this amounts to is:

'There will be an examination tomorrow, but you believe that there won't be.'

Now, as we have seen, there are peculiarities about the first-person version of such claims. In particular, if a pupil appropriates the teacher's announcement, and embraces it as his own in the form

'There will be an examination tomorrow, but *I* believe that there won't be.'

then, taking the second conjunct as the *expression* of a belief that there will not be an examination, we have a contradiction, from which it would be unwise for the pupil to infer anything and, in particular, to infer that there will be no examination tomorrow. We saw, however, that, in another sense, to ascribe a belief is to ascribe a mental state, and, if the pupil reads the teacher's announcement in that sense, he will see it merely as a prediction about his (the pupil's) state of mind, a prediction that may be *wrong* even though, in the past, most of the teacher's similar predictions have been right. This possibility, once countenanced, should prevent the pupils argumentativly proceeding to their over-optimistic conclusion.

Third Transition – From Catch-22 to the Liar

When we looked hard at the Barber and the Russell Paradox, we observed that what at first sight was a description of a barber and a specification of a set turned out, in each case to be a contradiction which describes or specifies nothing.. In Joseph Heller's novel *Catch-22*, there is a

clause which seems to specify the conditions under which an airman can be excused combat duty. But there is a catch – it is a condition that cannot be satisfied:

> 'You mean there's a catch?'
>
> 'Sure there's a catch,' Doc Daneeka replied. 'Catch-22. Anyone who wants to get out of combat duty isn't really crazy.'
>
> There was only one catch and that was Catch-22, which specified that a concern for one's own safety in the face of dangers that were real and immediate was the process of a rational mind. Orr was crazy and could be grounded. All he had to do was ask; and as soon as he did, he would no longer be crazy and would have to fly more missions. Orr would be crazy to fly more missions and sane if he didn't, but if he was sane he had to fly them. If he flew them he was crazy and didn't have to; but if he didn't want to he was sane and had to. Yossarian was moved very deeply by the absolute simplicity of this clause of Catch-22 and let out a respectful whistle.
>
> 'That's some catch, that Catch-22,' he observed.
>
> 'It's the best there is,' Doc Daneeka agreed.[6]

It looks as if an airman can get out of flying dangerous missions on condition that he is insane, for we have

1. Anyone can avoid flying missions if and only if he is insane

All you need do is to establish your insanity. Now, it defines you as being insane if you *don't* ask to be spared flying missions:

2. Anyone is insane if and only if he does not request to be taken off missions.

But you cannot be spared flying missions unless you request it:

3. Anyone who does not request it cannot avoid flying missions

Now, 1.,2. and 3. jointly entail

4. Anyone can avoid flying missions if and only if he cannot avoid flying missions

So we end up not with the condition one has to meet in order to avoid flying missions, but merely with a contradiction which specifies no condition at all. Notice that this is not the same as a condition that cannot be satisfied, such as `You can avoid flying missions if and only if you can trisect an arbitrary angle using only straightedge and compass'; it just does not amount to the expression of any condition at all.

In order to make the transition to the Liar Paradox, consider a statement `S is not true', where S is the name of that very statement. So here we have a statement that says of itself that it is not true. What would things have to be like for S to be true? Well, consider that question raised about a non-problematic statement like `On Monday, November 29, Laurence is lecturing in the Taliesin'. Call that statement `A'. The answer to the question of how things have to be like for A to be true is simple:

A is true if and only if on Monday, November 29, Laurence is lecturing in the Taliesin.

I have just given what are called the truth-conditions for statement A. But now, if we employ the same technique for giving the truth-conditions for S we get:

S is true if and only if S is not true.

Sound familiar? It's like Catch-22 all over again. And just as, in that case, no condition was specified for avoiding flying missions, so here no *statement* is specified – there just is no statement S which could be both true and not true. We can prove this in a slightly more convoluted way: Could `S' be the name of the statement `S is not true'? If we assume that `S' names a *true* statement, then it obviously cannot be the name of the statement `S is not true', for the latter would (on the covering assumption) be *false*. On the other hand, if we assume that `S' names a *false* statement then it obviously cannot be the name of the statement `S is not true', for the latter would (on the covering assumption) be *true*. So `S' cannot be the name of the statement that S is not true – in other words, there can be no statement which says of itself that it is not true.

To which conclusion, the exasperated reaction might be: `That's just not true'. For can't we simply refute the conclusion by coming right out with a statement that *does* state of itself that it is not true, namely

This statement is not true

? It is at this point that we need to move up a philosophical gear, for the distinctions we need to draw are quite subtle. What I have just written (`This statement is not true') is certainly a *sentence*: it consists of yeoman words strung together in conformity with the grammatical rules of standard English. But that it is a sentence is no guarantee that it is, or that it yields, a *statement*. A statement is something that is either true or false. Typically, we use sentences to make statements and one physical sentence can be used to make *different* statements. For example, there is a sign in Swansea market which reads `There are pickpockets in this area'. I steal the sign, and, late at night, screw it to the door of the Llandovery monastery. Same sign, but different statements – one true, the other false.

What statement, if any, is made, is a function not just of the sentence used, but also of the context in which it is used, for the context delivers what the statement is about, and it is the possession of this quality of aboutness (sometimes called `intentionality') that distinguishes a statement from a syntactically correct blob.[7]

If I say `This number is greater than nine' and I am indicating the number seven, then I succeed in making a statement, but a false one. If I utter the same sentence, but there is no number in the physical or conversational vicinity then my sentence, though syntactically correct, fails to yield a statement, and the phrase `this number' is about no number. Similarly, if I say `That statement is true' and the statement I am referring to is the first one that the Vice-chancellor made in his introductory remarks this evening, then I succeed in making a statement, one which inherits its content from his. I have argued that the Liar sentence fails to yield a statement. If correct, this would mean that the paradox, which starts with the assumption that the Liar statement is either true or false, cannot even get started.

Fourth Transition – From the Infinite to the Finite

The Russell paradox speaks of classes which are members of themselves. The Liar paradox concerns a statement putatively about itself. Another paradox due to Kurt Grelling deals with predicates that are not true of themselves. As can be seen, reflexivity is present in all these paradoxes and it used to be thought that this property was a necessary feature of all paradoxes of in this family. However, in recent years, there have been discovered paradoxes both within semantics and set theory which are non-reflexive. These paradoxes are infinitary. It would be very nice if we could show a transition from the infinite to the finite

so that, if we have succeeded in showing that there is a thread connecting the shallow to the deep, extending it to the infinitary paradoxes will deliver an appealingly unified account.

One particularly interesting infinite chain was recently investigated by Stephen Yablo. It begins

(Y_1) For all k>1, (Y_k) is not true.

(Y_2) For all k>2, (Y_k) is not true.

(Y_3) For all k>3, (Y_k) is not true.

and has as its ith element:

(Y_i) For all k>i, (Y_k) is not true.

It is easy to show that this sequence generates a contradiction, and the same is true of a similar chain with each `all' replaced by a `some'. Moreover, for both of these semantical paradoxes there are corresponding set-theoretical ones, infinitely long.

The technique for making the transition from infinite to finite invokes the notion of a circular queue.[8] Let us take Yablo's Paradox as our exemplar. Replace the infinite sequence of statements with an infinite queue of people, each of whom says `Every person behind me in the queue is speaking an untruth'. Consider now a *finite* Yablo queue containing k queuers. This is unparadoxical, for, working backwards from the end, we can establish for each queuer whether what he or she says is true or false. But now imagine that we pull the tail of this queue round to the head, thus creating a finite, endless circular queue. In some queer sense, any queuer is k places behind *himself*, and paradoxicality is restored.

For a very tight circle consisting of just two queuers, the resulting paradox consist of the two statements:

($YC_{2(1)}$) Both of our statements are not true

($YC_{2(2)}$) Both of our statements are not true.

It may be easily verified that if either statement is true, it must be not true and if not true, then true. Similarly for the two-person `some' version which consists of two statements:

($SC_{2(1)}$) At least one of our satements is not true

($SC_{2(2)}$) At least one of our statements is not true

which is closely similar to one of the paradoxes studied by the mediaeval logician Jean Buridan. Shrink the circle still further, and the result is a one-person circle, with the individual in question uttering:

(SC_1) This statement is not true

So we are back again with the Liar Paradox! Another way of getting to the same point would be to start with an infinite queue, each individual uttering the sentence `The next statement is false' Here content is not inherited but deferred *ad infinitum.* Converting to a one-element circle produces the Liar, and reinforces the claim argued for previously that this is a sentence that fails to yield a contentful statement.

Transition pending – From the Better Lover to MINIAC

In the previous sections, I have tried to show how to move from one problem to another, so as to make plausible the claim that the solution to one is basically all that is required for solving the other. In this final section, I do not forge such a transition but feel in my bones that it is there to be forged, and I leave the forging as a task for the audience. The `shallower' problem in this case, may be called the `Better Lover' problem. Naturally, when two men are sharing one woman, each wants to know whether or not he is the better lover. He can ask, but there's no guarantee that he will get the right answer, since the woman may be a liar. Is there a single question that a man can ask so that he can find out whether he is or is not the better lover, even though the woman may be a habitual liar? The answer is `Yes' – he must ask the rather complicated question `When the other guy asked you whether I was the better lover, was your answer to him `Yes'?' If the woman is a truth-teller and answers `Yes', this means that her answer to the other guy was `yes', which means that the questioner (call him Sam) is the better lover. Suppose, however that the woman is a habitual liar. If she answers `Yes', that means that her answer to the other guy was `No' and, since that too was a lie, the truth is that Sam is the better lover. So, whether the woman is a truth-teller or an habitual liar, her answer of `Yes' to Sam's question establishes that Sam is the better lover. Likewise, if her answer to Sam's question is `No', then, whether the woman is a truth-teller or an habitual liar, it follows that Sam is not the better lover. This answer is correct – there is no paradox – the example is of the `brain-twister' kind that greatly exercised the mathematician Raymond Smullyan.

Contrast this non-paradox with the paradox of MINIAC, the world's smallest electronic brain:[9]

Take a coin and designate one side `Yes' and the other `No'. Think of a question for which you would very much like an answer, e.g. `Will there be peace in Northern Ireland within the next six months?'. Toss the coin and note the answer it gives. But how can we tell whether this answer is true or false? Easy. Ask the question `Will your present answer have the same truth-value as your previous answer?', flip the coin and note the response (`Yes' or `No'). If the second response is `Yes' then MINIAC's answer to the first question was true; if the second response is `No' then the answer to the first question was false, so now you know for sure there will be peace in Northern Ireland within the next six months Proof: (I'll just do the proof for the case where the second answer is `No.') Suppose the second answer is `No'. This answer must be true or false. If it's *true* then the answer to the first question is *false*. But, if the answer (`No') to the second question is *false*, then the truth-value of the second question must be the same as that of the first, so, again the answer to the first question is *false*. Therefore, if the second answer is `No' we have proved that, whether or not that answer is true, the answer given by MINIAC to the first question *must* be false. By similar reasoning, we can prove that, if MINIAC answers `Yes' to the second question, its answer to the first question must have been *true*.

What is paradoxical here, of course, is that one *cannot* guarantee to get correct answers to momentous questions merely from two flips of a coin. Obviously not – but why?.... what is wrong with the reasoning that has led to this blatantly absurd conclusion? On that question, I end.

[1] J. Bernadeté, *Infinity: an essay in Metaphysics* (Oxford, Clarendon Press, 1964), p.259. For a recent discussion of Bernadeté's paradox, see Graham Priest, `On a version of one of Zeno's paradoxes', *Analysis* 59 (1999), pp.1-2.

[2] I have attempted to do just this in my `Escher, Reflexivity, Contradiction and Paradox', *Leonardo 1996.*

[3] For a proof of this point, see my `A Unified Solution to Some Paradoxes', *Proceedings of the Aristotelian Society, New Series* Vol.C (2000), pp.53-74.

[4] For an amusing investigation into this question, see Roy Sorensen, `Faking Munchausen's Syndrome' (forthcoming)

[5] Leslie Stevenson, `First Person Epistemology', *Philosophy* 74 (1999), 475-97; see p.479.

[6] *Joseph Heller, Catch-22* (London, Vintage, 1994), pp.62-63. The book was first published in 1961. For a comparison between Catch-22 and the paradox of Protagoras and Eulathus, see William Poundstone, *Labyrinths of Reason* (London, Penguin, 1998), p.128.

[7] Simon Blackburn makes a similar point about thoughts. A thought is not a blob in the brain, even though that blob might be the inscription of a sentence. See his *Think* (Oxford, Oxford University Press, 1999), p.79.

[8] A fuller discussion is contained in my `Circular queue paradoxes – the missing link', forthcoming in *Analysis*.

[9] T. Storer, `MINIAC: World's Smallest Electronic Brain', *Analysis* 22 (1961-2), pp.151-152.